SUBJECT: MATHEMATICS

PAPER: *LINEAR PROGRAMMING*

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Syllabus

Linear Programming

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Unit-I

Linear programming problems, Statement and formation of general linear programming problems, Graphical method, Slack and surplus variables, Standard and matrix forms of linear programming problem, Basic feasible solution.

Unit-II

Convex sets, Fundamental theorem of linear programming, Simplex method, Artificial variables, Big-M method, Two phase method.

Unit-III

Resolution of degeneracy, Revised simplex method, Sensitivity Analysis.

Unit-IV

Duality in linear programming problems, Dual simplex method, Primal-dual method Integer programming.

Unit-V

Transportation problems, Assignment problems. Goal Programming; Concept of goal programming, formulation and methodology for solution of goal programming.

Lecture - 1

LINEAR PROGRAMMING PROBLEM

Linear Programming :- Linear programming is an important optimization (maximization or minimization) technique used in decision making in business and every day life for obtaining the maximum or minimum values as required of a linear expression subject to satisfying certain number of given linear restrictions.

Linear Programming Problem (L.P.P) :- The LPP in general calls for optimizing (maximizing or minimizing) a linear function of variables called the objective function subject to a set of linear equations and/or linear inequalities called the restrictions or constraints.

Linear → means all the relations governing the problem are linear.

Programming → means the process of determining a particular programme or plan of action.

objective function :- The function which is to be optimized (maximized or minimized) is called an objective function.

Constraints :- The system of linear inequations (or equations) under which the objective function is to be optimized are called the constraints.

Mathematical Description of a General linear Programming Problem :-

A general LPP can be stated as follows :

find $x_1, x_2, --- x_n$ which optimize the linear function

$$Z = c_1 x_1 + c_2 x_2 + --- + c_n x_n \quad \text{--- (1)}$$

subject to the constraints

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + ------ + a_{1n}x_n \ (\leq = \geq)\ b_1 \\ a_{21}x_1 + a_{22}x_2 + ------ + a_{2n}x_n \ (\leq = \geq)\ b_2 \\ ------------ \\ ------------ \\ a_{m1}x_1 + a_{m2}x_2 + ------ + a_{mn}x_n \ (\leq = \geq)\ b_m \end{array}\right] \text{--- (2)}$$

and the non-negative restrictions

$$x_1, x_2, ----, x_n \geq 0 \quad \text{--- (3)}$$

where all $a_{11}, a_{12}, ---- a_{mn}$; $b_1, b_2 ----, b_m$; $c_1, c_2, --- c_n$ are constants and $x_1, x_2, --- x_n$ are variables.

The function Z is called objective function

The conditions given in (2) are called linear Constraints and the conditions given in (3) are called the non- negative restrictions of the LPP.

The LPP may be stated in matrix form as follows:-

Find $x_1, x_2, --- x_n$ so as to optimize

$$Z = cx$$

$$Z = Cx$$

subject to $\quad Ax \ (\leq = \geq) \ b$

and $\quad x \geq 0,$

where $\quad A = [a_{ij}]_{m \times n} \longrightarrow$ Coefficient matrix

$C = [c_1, c_2, \cdots c_n] \longrightarrow$ price vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = [b_1 \ b_2 \cdots b_m]' \longrightarrow \text{Requirement vector}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_1 \ x_2 \cdots x_n]' \longrightarrow \text{matrix of Variables}$$

$0 \rightarrow$ null matrix of type $n \times 1$

Working Rule to form a linear Programming Problem :-

(1) Identify the variables in L.P.P. and denote them by x_1, x_2, x_3 etc.

(2) Identify the objective function and express it as a linear function of variables x_1, x_2, x_3 etc.

(3) Find the type of the objective function (maximizing Profits or minimizing cost)

(4) Identify all the constraints and express them as linear inequations or equations.)

Example ***A goldsmith manufactures necklaces and bracelets. The total number of necklaces and bracelets that he can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. It is assumed that he can work for a maximum of 16 hours a day. Further the profit on a bracelet is ₹ 300 and the profit on a necklace is ₹ 100. Formulate this problem as a linear programming problem so as to maximize the profit.***

Solution: Suppose the goldsmith manufactures x_1 necklaces and x_2 bracelets per day.

Since the profit on a necklace is ₹ 100 and profit on a bracelet is ₹ 300, therefore the total profit Z in ₹ is given by

$$Z = 100x_1 + 300x_2 \qquad \ldots(1)$$

Since it takes half an hour to make one necklace, so the time required to make x_1 necklaces = $(1/2)x_1$ hours.

Again it takes one hour to make one bracelet, so the time required to make x_2 bracelets

$$= 1.x_2 \text{ hours} = x_2 \text{ hours.}$$

Therefore, total time required to make x_1 necklaces and x_2 bracelets

$$= (x_1/2 + x_2) \text{ hours.} \qquad \ldots(2)$$

Since total time available per day is 16 hours, therefore

$$x_1/2 + x_2 \leq 16 \quad \text{or} \quad x_1 + 2x_2 \leq 32. \qquad \ldots(3)$$

The total number of necklaces and bracelets that the goldsmith can manufacture in a day is atmost 24, so we have

$$x_1 + x_2 \leq 24. \qquad \ldots(4)$$

Also the number of necklaces and bracelets manufactured can never be negative, therefore

$$x_1 \geq 0, x_2 \geq 0. \qquad \ldots(5)$$

Hence, the linear programming problem formulated from the given problem is as follows:

$$\text{Maximize } Z = 100x_1 + 300x_2$$

subject to the constraints

$$x_1 + 2x_2 \leq 32$$

$$x_1 + x_2 \leq 24$$

and the non-negative restrictions $x_1 \geq 0, x_2 \geq 0$.

Example *According to the medical experts it is necessary for an adult to consume at least 75 gms of proteins, 85 gms of fats and 300 gms of carbohydrates daily. The following table gives the analysis of the food items readily available in the market with their respective costs.*

Food Type	Food value (in gm.) per 100 gms			Cost in ₹ per kg.
	Proteins	Fats	Carbohydrates	
A	18.0	15.0	—	3.0
B	16.0	4.0	7.0	4.0
C	4.0	20.0	2.5	2.0
D	5.0	8.0	40.0	1.5
Minimum daily requirement	75.0	85.0	300.0	

Formulate a linear programming problem for an optimum diet.

Solution: Let the daily diet consist of x_1 kg. of food A, x_2 kg. of food B, x_3 kg. of food C and x_4 kg. of food D.

Then the total cost per day in ₹ is

$$Z = 3x_1 + 4x_2 + 2x_3 + 1.5x_4. \qquad ...(1)$$

Total amount of proteins in the daily diet is

$$(180x_1 + 160x_2 + 40x_3 + 50x_4)$$

Since the minimum daily requirement of proteins is 75 gms, therefore we have

$$180x_1 + 160x_2 + 40x_3 + 50x_4 \geq 75 \qquad ...(2)$$

Similarly, considering the total amounts of fats and carbohydrates in the diet, we have

$$150x_1 + 40x_2 + 200x_3 + 80x_4 \geq 85 \qquad ...(3)$$

and

$$70x_2 + 25x_3 + 400x_4 \geq 300. \qquad ...(4)$$

Since, the daily diet cannot contain quantities with negative values of any food item, therefore

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0. \qquad ...(5)$$

Hence, the linear programming problem formulated for the given diet problem is :

$$\text{Minimize } Z = 3x_1 + 4x_2 + 2x_3 + 1.5x_4$$

subject to the constraints

$$180x_1 + 160x_2 + 40x_3 + 50x_4 \geq 75$$

$$150x_1 + 40x_2 + 200x_3 + 80x_4 \geq 85$$

$$70x_2 + 25x_3 + 400x_4 \geq 300$$

and the non-negative restrictions

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \text{ and } x_4 \geq 0.$$